Hedging and Speculation with Bitcoin Derivatives

Frontiers in Finance

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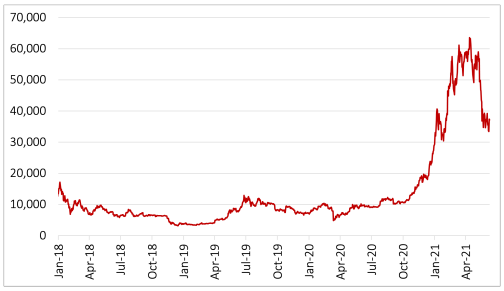
Outline

1. Introduction

Bitcoin Spot Price: 1 Jan 2018 to 10 June 2021



Who Speculates on Bitcoin and Where?

- Traditional retail investor base
  - Reddit
  - Robinhood
- More recently, institutional investors
  - US banks allowed custody of bitcoin since June 2020
  - Specialised traders and brokers – GSG, XBTO, BitWise etc.
- Which derivatives on which exchanges have most speculation?
  - Diverse fee structure on different exchanges ⇒ specialised traders
    - E.G. Binance attracts large positions (BNC rewards)
    - Bybit fee rebates encourage wash traders
  - CME futures follow price discovery on unregulated exchanges (Alexander and Heck, 2020)

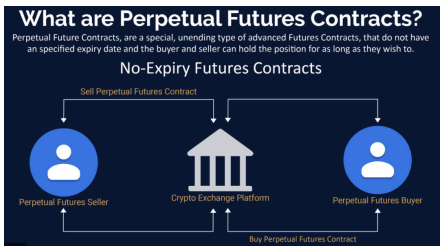
Who Hedges Bitcoin?

- Bitcoin Miners:
  - About 18.5 million coins have been mined
  - BTC supply stops in 2140, capped at 21 million
  - Currently, 6.25 coins mined every 10 mins
  - About 150,000 BTC mined since 1 Jan 2021
  - 150,000 BTC worth almost $10 trillion at height of market
- Option Traders:
  - Open interest on Deribit bitcoin options currently ∼ $6 billion
  - Relative volumes ⇒ market makers delta hedge with perpetual futures

Standard, Direct and Inverse Bitcoin Futures

- Standard bitcoin futures
  - Bitcoin is the underlying asset
  - E.G. CME contracts have notional value 5 or 0.1 BTC
  - Quoted, margined and settled in USD
  - Standard expires, but do not trade 24/7
- USDT direct futures
  - Bitcoin is the underlying asset
  - E.G. Bybit direct USDT futures have notional value of 1 BTC
  - Quoted, margined and settled in a stablecoin like USDT
  - Non-standard expires, traded 24/7
- USD inverse futures
  - Quoted in USD per bitcoin, like direct futures
  - But with USD notional and margined and settled in BTC
  - E.G. Binance inverse futures have a notional value of $100
  - Non-standard expires, traded 24/7

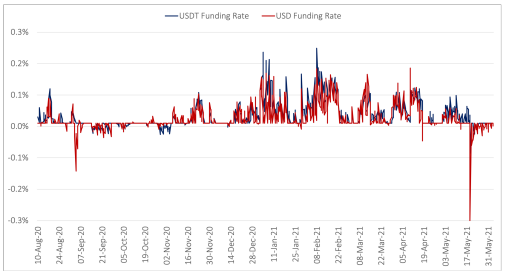
Perpetual Futures Contracts



Introduction to Binance Perpetuals

Funding Rate Payments

Funding Rates for Binance USDT Direct Perpetual

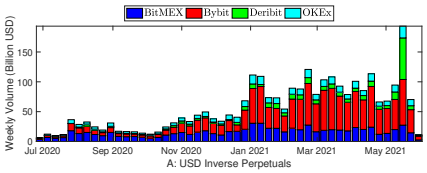


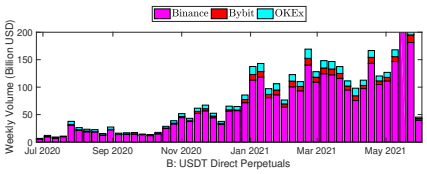
Funding Rate Calculation

Which Product Type and Exchange is Best for Hedging?

Three Reasons Why

Trading Volumes on Main Exchanges





Binance USDT Direct Perpetual

125x USDT-Margined Perpetual Contract (BTCUSDT)

Position (Notional Value in USDT)	Leverage	Initial Margin Rate
0 < Position ≤ 50,000	125x	0.80%
50,000 < Position ≤ 250,000	100x	1.00%
250,000 < Position ≤ 1,000,000	50x	2.00%
1,000,000 < Position ≤ 10,000,000	20x	5.00%
10,000,000 < Position ≤ 20,000,000	10x	10.00%
20,000,000 < Position ≤ 50,000,000	5x	20.00%
50,000,000 < Position ≤ 100,000,000	4x	25.00%
100,000,000 < Position ≤ 200,000,000	3x	33.30%
200,000,000 < Position ≤ 300,000,000	2x	50.00%
300,000,000 < Position ≤ 500,000,000	1x	100.00%

- Maximum leverage decreases with position size
- Maintenance margin rate is half the initial margin rate

Contract Specifications

2. Auto-Liquidations

- Unregulated exchanges issue no margin calls
  - Liquidations are automatically triggered when marked loss exceeds the collateral in the margin account
- Settlements are based on an index price – usually a simple average of BTC rates across the main spot exchanges
  - Many exchanges employ a fair price marking mechanism to mitigate auto-liquidations triggered by sudden spikes in index prices:

$$\text{Fair Mark Price} = \text{Index Price} \times (1 + \text{Funding Basis})$$
$$\text{Funding Basis} = \text{Funding Rate} \times (\text{Time to Funding} \,/\, \text{Funding Interval})$$

  - Fair mark price is only used for unrealised P&L
- Funding rates are too small to impact the frequency of auto-liquidations
  - Except in crashes like 12-13 March 2020 or 11-19 May 2021

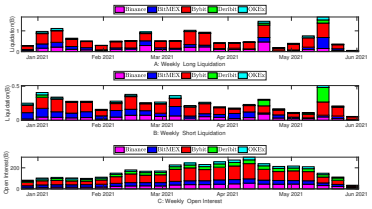
Historical Frequency of Auto-Liquidation

- Suppose trader opens 1 long or short position, so max. leverage is 100X
- Suppose leverage is 5X, 20X, 50X or 100X
- So initial margin rate is 20%, 5%, 2% or 1%
- Assume maintenance margin rate is half the initial margin
- What is probability of auto-liquidation if position is held for 8h – 30d?

	Leverage	8h	1d	5d	15d	30d
Long	5X	0.25%	1.34%	7.49%	15.38%	22.29%
	20X	18.84%	35.56%	56.48%	65.12%	66.17%
	50X	45.05%	61.04%	76.98%	82.16%	82.43%
	100X	64.19%	75.57%	85.12%	87.89%	87.94%
Short	5X	0.65%	4.95%	31.34%	60.44%	80.98%
	20X	20.62%	41.73%	75.14%	91.00%	96.51%
	50X	49.71%	70.35%	89.19%	96.35%	98.17%
	100X	69.41%	83.39%	93.48%	97.71%	98.85%

Bybit USD Inverse Perpetuals 1-min data, 1 July 2020 to 31 May 2021

Auto-Liquidations on USD Inverse Perpetuals ($Billion)

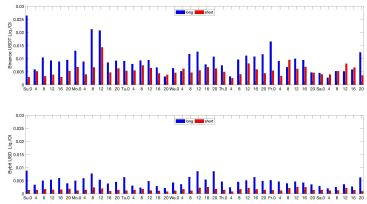


4-hourly data from coinanalyse.net

- Auto-liquidations are greatest on Bybit, but so is open interest
- BitMEX & Deribit also forced abnormal liquidations between 11 and 19 May
- Deribit's liquidations ⇐ market makers using perpetual to delta hedge options

Time Pattern of Auto-Liquidations?

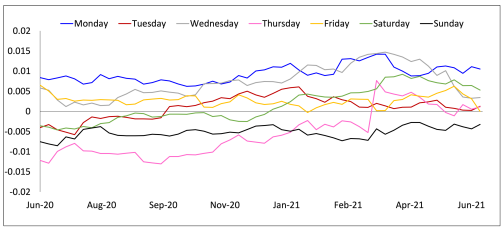
Auto-liquidations as % OI on Binance and Bybit in 4hr time buckets



- Pattern represents period between 1 Jan to 31 May 2021
- Long auto-liquidations usually exceed short
- Binance is the most aggressive auto-liquidator
- Greatest amount of liquidations Sun 00:00 – 08:00 and Mon 08:00 – 16:00

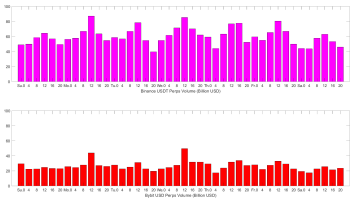
Day-of-Week Effects in Returns

52-week moving average of returns on different days



- Sunday average returns are negative ~ -5 bps
- Monday average returns are positive $\sim +10$ bps

Time Patterns in Traded Volume



- Neither auto-liquidations nor day-of-week effects associated with lower volume
- Volume not lower at weekends ⇒ large retail base
- Time pattern of trading pattern on Binance ⇒ US investors
- Less obvious on Bybit

3. Volume and Volatility Spillovers

- Numerous recent papers on bitcoin price discovery
  - Most focus on the role of CME futures
- Instead, Alexander and Heck (2020) and Alexander et al. (2020b) highlight the role of unregulated exchanges such as BitMEX
  - And Alexander et al. (2020a) examines price discovery in ether
- But crypto markets change extremely rapidly
  - Not only through innovations such as DeFi and NFTs
  - But also due to actions of US regulators
  - And the Chinese government ... and Elon Musk
- Price and volatility transmissions now mostly from the Binance Asia exchange
  - And the most influential Binance product is its USDT perpetual
  - Is trading on this product driving tether grants?
  - Tether market cap is now $62 billion and rising!
- Work in progress

High-Frequency Volatility Spillover

Exchanges

Spot	Perpetuals
Bitstamp$^{\$}$	Binance$^{\$}$
Coinbase$^{\$}$	Bybit$^{\$}$
Kraken$^{\$}$	Binance^{T}
Binance^{T}	
Huobi^{T}	

Research Questions

- Where does bitcoin volatility emerge?
  - Which exchange is the main emitter?
- To which exchanges does volatility transmit?
  - Which exchanges receive the most?
- Do volatility flows change over the course of the day?

Perpetual Specifications and Data

	USD Contracts		USDT Contracts
	Binance	Bybit	Binance
Type	Inverse	Inverse	Direct
Contract Size	100 USD	1 USD	0.001 BTC
Initial Margin Rate	$>$ 0.8%*	1%	$>$ 0.8%*
Settlement Currency	BTC	BTC	USDT
Trading Days	24/7	24/7	24/7
Funding Frequency	8 hrs	8 hrs	8 hrs
Fees (maker/taker) in bps	1/5	-2.5/7.5	2/4
Tick Size	0.1 USD	0.5 USD	0.01 USDT

Note: Only showing the main perpetual contracts analysed. *Margin rates on Binance increase with notional value of position.

- Second-by-second data from 1 January to 31 March 2021
- Calculate realised volatility at 5-minute frequency
- Winsorizing top 0.05% to reduce influence of extreme outliers
- Pre-averaging to reduce microstructure noise

Intra-day Volume Patterns

Note: The figure shows the intraday pattern of five-minute trading volume (in million USD) for USD spot pairs (upper graph), USDT-spot pairs (middle graph) and perpetuals (lower graph), measured as the median five-minute volume over the period from 1 January to 31 March 2021. All times are in UTC.

- Coinbase trading similar to FX markets $\rightarrow$ increased volume between 12:00 UTC (7:00 EST) and 21:00 UTC (16:00 EST)
- Spikes at funding times of perpetuals, especially midnight and 16:00 UTC

Intra-day Realised Volatility Patterns

Intra-day pattern removed before model estimation

Note: The figure shows the intraday pattern of 5-minute realised volatility (in million USD) for USD spot pairs (upper graph) as well as USDT spot pairs and perpetuals (lower graph), measured as the average five-minute realised volatility over the period from 1 January to 31 March 2021. All times are in UTC.

Vector Logarithmic Multiplicative Error Model

Basic specification for 5-min realised volatilities of 6 exchanges, $\boldsymbol{x}_t$:

$$\boldsymbol{x}_t = \boldsymbol{\mu}_t \odot \boldsymbol{\varepsilon}_t$$
$$\log \boldsymbol{\mu}_t = \mathbf{w} + \mathbf{A} \log \boldsymbol{x}_{t-1} + \mathbf{B} \log \boldsymbol{\mu}_{t-j}$$

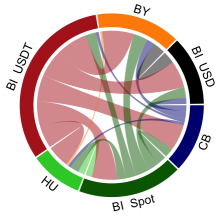
- Implicitly guarantees non-negativity of realised volatilities
- Decomposes realised volatility into Hadamard product of conditional mean and error term with unit mean and a distribution with non-negative support
- Log conditional mean is autoregressive $\Rightarrow$ long-term effects **B**
- Dependence on lagged observations $\log \boldsymbol{x}_{t-1} \rightarrow$ short-term spillovers, **A**
- Add asymmetric response component to capture leverage effect
- Also use an extension to capture zeros in high-frequency time series
- Also use dummies to investigate time-zone effects

Results – Main Instruments

	Coinbase	**Binance^{S}**	**Huobi**	**Binance^{T}**	**Bybit**	**Binance$^{\$}$**
CB	0.2108	-0.0895	$0.0352^{n.s}$	0.2548	$0.0088^{n.s}$	$-0.0173^{n.s}$
BI^{S}	$-0.0164^{n.s}$	0.0742	0.0626	0.2439	$0.0105^{n.s}$	$0.0223^{n.s}$
HU	-0.0282	-0.0851	0.2720	0.2177	0.0198	$-0.0108^{n.s}$
BI^{T}	-0.0307	-0.0995	$0.0247^{n.s}$	0.4702	$0.0149^{n.s}$	$0.0132^{n.s}$
BY	-0.0761	-0.1373	$0.0441^{n.s}$	0.3085	0.1450	0.1200
BI$^{\$}$	-0.0594	-0.1039	$0.0110^{n.s}$	0.2606	$0.0134^{n.s}$	0.2742

- Parameter estimates for matrix **A** of the multivariate LogMEM(1,1)$_1$, fitted to 5-min realised volatility on Coinbase (CB), Binance Spot (BI^{S}), Huobi (HU), Binance USDT-perpetual (BI^{T}), Bybit (BY) and Binance USD-perpetual (BI$^{\$}$)
- Column denotes emitting exchange, row denotes receiving exchange
- Diagonals in red capture flows back into exchange
- Superscript $^{n.s}$ indicates estimate is not significant at 1%
- Data period 1 January to 31 March 2021.

Answers to Research Questions



- Where does bitcoin volatility emerge?
  Mostly from Binance Asia
- To which exchanges does volatility transmit?
  Bybit and the spot exchanges Coinbase and Binance US
- Do volatility flows change over the course of the day?
  Yes, they increase at time of funding payments on perpetuals

4. Optimal Hedging in the Presence of Auto-Liquidations

Research Questions

1. Which product on which exchange is best for hedging?
2. Assuming you use the best product, what is the optimal hedge ratio that also minimizes the probability of auto-liquidations?
3. And, given your level of loss aversion, how much collateral should you keep in the margin account to achieve, say, 95% hedging effectiveness?

Problem Formulation

- Suppose the platform monitors the margin at discrete intervals Δt
- Assume the hedger holds 1 bitcoin at time t with spot price S_t and shorts θ units of the futures to hedge spot volatility until hedge horizon $t + N\Delta t$
- Direct and inverse perpetual futures prices: F_t and $\widehat{F}_t = F_t^{-1}$
- Set n-period profit from long position $\Delta_n S_t := S_{t+n\Delta t} - S_t$,
- Similarly, n-period loss on short position is $\Delta_n F_t := F_{t+n\Delta t} - F_t$
- Set $\Delta_n \widehat{F}_t = \widehat{F}_t - \widehat{F}_{t+n\Delta t}$ (in BTC) so a short position makes losses from an increase in its USD value
- Need to multiply $-\theta\, \Delta_n \widehat{F}_t$ by $S_{t+n\Delta t}$ to convert into USD
- Choose θ to minimize the variance of the P&L of the hedged portfolio:

$$\sigma_h^2(\theta) = \begin{cases} \mathbb{V}\mathrm{ar}\left(\Delta_N S_t - \theta\, \Delta_N F_t\right) & \text{(Direct)} \\ \mathbb{V}\mathrm{ar}\left(\Delta_N S_t - \theta\, \Delta_N \widehat{F}_t \cdot S_{t+N\Delta t}\right) & \text{(Inverse)} \end{cases},$$

Adding the Margin Constraint

- Following Longin (1999), introduce the extreme price change operators:

$$\Delta_N^* F_t := \max_{1\le n\le N} \Delta_n F_t, \quad \Delta_N^* \widehat{F}_t := \max_{1\le n\le N} \Delta_n \widehat{F}_t$$

- Since we are short, a positive value implies a loss
- Margin constraint is represented by another parameter $m \geq 0$ being the collateral in the margin account, in excess of the minimum maintenance margin
- In other words, m is the upper constraint on the hedger's ability to avoid auto-liquidation
- Hence, probability of auto-liquidation is

$$P(m,\theta) = \mathbb{P}\mathrm{rob}(\text{auto-liquidation}) := \mathbb{P}\mathrm{rob}\left(\theta \cdot \Delta_N^* X_t > m\right)$$

  where X is price of either direct F or inverse $\hat{F}$ perpetual
- This probability decreases with m and increases with θ

Hedging Problem and Solution

Introduce a loss aversion parameter γ to obtain final objective:

$$\min_{\theta>0} \quad \left\{\sigma_h^2(\theta) + \gamma\, \sigma_S^2\, P(m,\theta)\right\}$$

The optimal hedging strategy θ^* is given by

$$\theta^* = \omega^{-1}\, \theta_0, \qquad \omega = \begin{cases} 1 & \text{(Direct)}, \\ \frac{S_t}{F_t^2} & \text{(Inverse)}. \end{cases}$$

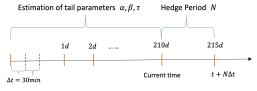
where θ_0 is a positive root of the equation $\alpha(x)x^{-2} + x - \beta = 0$ with:

$$\alpha(x) = \frac{\gamma m v^2 w}{2\zeta} \frac{\sigma_{\Delta S}^2}{\sigma_{\Delta F}^2} \exp\left[-\left(1 + \tau \frac{mwx^{-1} - \mu}{\zeta}\right)^{-\frac{1}{\tau}}\right] \left(1 + \tau \frac{mwx^{-1} - \mu}{\zeta}\right)^{-\frac{1}{\tau}-1}.$$

Extreme Value Distributions

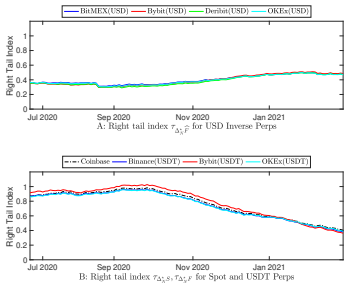
In addition to γ and m, the parameters of the optimal strategy are as follows:

- $v = \frac{\sigma_S}{\sigma_F}$ is relative volatility of n-period spot and futures P&L
- $\beta = \frac{\sigma_{SF}}{\sigma_F^2}$ where the numerator is their covariance
- The constants μ, ζ and τ are location, scale and right tail index parameter of distribution of extreme values of direct/inverse perpetuals – for short positions
- Illustrate their estimation for 30-minute monitoring and 5d hedge horizon
- GEV parameter estimates based on rolling sample of 210 daily extreme values:



- Most important parameter for optimal short hedge ratio is right tail index τ

Right Tail Index Estimations



A: Right tail index $\tau_{\Delta,F}$ for USD Inverse Perps

B: Right tail index $\tau_{\Delta,S}, \tau_{\Delta,F}$ for Spot and USDT Perps

Similarity of tail index for direct (USDT) perpetuals ⇒ these are better hedge?

Hedge Effectiveness – USD Inverse Perpetuals

		BitMEX			Bybit			OKEx			Deribit		
Collateral	Loss Aversion	8h	1d	2d	8h	1d	2d	8h	1d	2d	8h	1d	2d
0.1	40	49.9	23.6	12.6	49.6	24.8	12.2	47.3	22.7	12.7	49.9	22.7	9.0
	20	67.7	34.9	17.5	66.9	36.4	18.5	64.3	34.6	16.2	67.8	36.8	15.7
	10	83.4	51.8	27.8	82.9	51.1	28.2	80.2	50.4	26.5	83.4	52.1	28.9
0.2	40	84.5	55.1	30.9	83.8	54.8	32.3	82.2	53.5	30.7	84.7	55.2	30.3
	20	93.3	72.7	45.9	93.1	71.6	45.5	91.9	70.5	44.3	93.5	72.8	46.3
	10	97.4	86.6	64.7	97.4	85.9	64.4	96.7	84.6	62.2	97.7	86.6	64.1
0.5	40	98.8	93.8	78.2	98.8	93.1	78.4	98.7	92.6	77.4	99	93.8	76.9
	20	99.3	97.8	90.0	99.5	97.4	89.9	99.4	97	89.3	99.6	97.8	89.3
	10	99.4	99.3	96.3	99.7	99.1	96.1	99.7	98.8	95.9	99.7	99.3	96.0

- Effectiveness ↓ as hedging period ↑ and as loss aversion ↑
- Effectiveness ↑ with collateral in margin account
- Exchanges offer similar protection
  - Except Deribit for longer-term hedges for traders with high loss aversion
- EG. A 2d hedge requires 50% of notional as collateral for 95% effectiveness
  - But only possible with low levels of default aversion

Hedge Effectiveness – USDT Direct Perpetuals

		Binance			Bybit			OKEx		
Collateral	Loss Aversion	8h	1d	2d	8h	1d	2d	8h	1d	2d
0.1	40	76.7	44.2	31.9	73.9	42.3	16.1	76.5	44.3	31.8
	20	92.9	78.2	55.6	91.5	76.7	55.6	92.7	78.1	55.5
	10	97.8	92.5	85.5	96.9	92.2	85.5	97.8	92.5	85.4
0.2	40	94.8	84.9	62.4	93.4	82.6	61.7	94.7	84.9	62.3
	20	98.5	95.4	89.4	97.5	94.8	89.2	98.4	95.4	89.3
	10	99.5	98.7	97.0	98.7	98.5	96.9	99.4	98.7	97.0
0.5	40	99.2	98.2	94.7	98.3	97.7	94.7	99.2	98.3	94.7
	20	99.7	99.5	98.6	98.9	99.3	98.5	99.6	99.5	98.6
	10	99.8	99.8	99.6	99.1	99.7	99.6	99.7	99.8	99.6

- Effectiveness always greater than using inverse perpetual on same exchange
- The three contracts offer similar protection
- Examples for 99% effectiveness:
  - When $\gamma = 10$ an 8hr hedge requires 20% collateral
  - When $\gamma = 40$ an 8hr hedge requires 50% collateral ...
  - .... but a 2d hedge requires more than 50% collateral

Answers to Research Questions

1. Which product on which exchange is best for hedging?
   - Direct USDT perpetuals are more effective than inverse contracts
   - Binance, Bybit and OKEx offer similar protection
2. Assuming you use the best product, what is the optimal hedge ratio that also minimizes the probability of auto-liquidations?
3. If you are highly loss averse ($\gamma = 0.4$), how much collateral should you keep in the margin account for 95% hedging effectiveness?
   - An 8h hedges requires ∼ 20% collateral
   - A 24h or longer hedge requires ∼ 50% collateral
4. Or, if you are not very loss averse ($\gamma = 0.1$) how much collateral should you keep in the margin account for 95% hedging effectiveness?
   - An 8h hedge requires ∼ 10% collateral
   - A 24h or longer hedge requires ∼ 20% collateral

5. Speculation

Turnover Index Garcia et al. (1986)

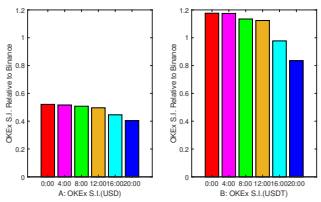
$$\text{TI} = \frac{\text{Trading Volume}}{\text{Open Interest}}$$

		USD Inverse Perpetuals					USDT Direct Perpetuals		
		Binance	BitMEX	Bybit	Deribit	OKEx	Binance	Bybit	OKEx
Mean	Volume (B)	1.03	0.49	1.20	0.20	0.25	2.75	0.22	0.29
	OI (B)	0.76	0.73	1.85	0.58	0.36	1.54	0.29	0.14
	Short Liq. (M)	1.05	1.48	2.87	0.48	0.39	7.90	1.50	1.54
	Long Liq. (M)	3.23	2.80	8.64	0.55	0.92	13.92	2.05	1.23
	Liquidation (M)	4.28	4.28	11.51	1.03	1.31	21.83	3.55	2.77
	TI	1.54	0.71	0.77	0.38	0.76	1.91	0.80	2.11
Median	Volume (B)	0.88	0.40	0.95	0.15	0.20	2.31	0.17	0.23
	OI (B)	0.70	0.75	1.86	0.58	0.34	1.61	0.30	0.14
	Short Liq. (M)	0.59	0.32	1.69	0.01	0.14	3.93	0.72	0.50
	Long Liq. (M)	1.23	0.37	4.08	0.02	0.16	5.08	1.05	0.49
	Liquidation (M)	2.07	1.33	6.72	0.05	0.52	11.67	2.34	1.45
	TI	1.24	0.57	0.58	0.28	0.60	1.54	0.62	1.66

The 4-hour mean and median of trading volumes and open interest (billion USD), and short, long and total liquidations (million USD). Bitcoin perpetuals are USD inverse and USDT direct across Binance, BitMEX, Bybit, Deribit and OKEx. The data period is from 1 Jan to 31 May 2021. Data acquired from www.coinalyze.net.

Periodicity in OKEx Turnover Index

- Turnover on OKEx, relative to Binance, for six 4-hour internals
- Data averaged over period 1 Jan to 31 May 2021



- Inverse perpetual (USD) turnover smaller than direct perpetual (USDT)
- Highest turnover 00:00 – 08:00 UTC [08:00 – 16:00 in China]
- Lowest turnover volumes 16:00 – 24:00 UTC [00:00 – 08:00 in China]

Auto-Liquidation Index

$$\text{LI} = \text{Liquidation Index} = \frac{\text{Liquidation Volume}}{\text{Trading Volume}}$$

- Numeraire is total volume of auto-liquidations, can be measured every 4 hrs
- Also decomposed into long and short volumes of auto-liquidations

Average of 4-hourly Liquidation Indices between 1 Jan and 31 May 2021

	USD Inverse Perpetuals					USDT Direct Perpetuals		
	Binance	BitMEX	Bybit	Deribit	OKEx	Binance	Bybit	OKEx
LI (long)	0.11%	0.34%	0.55%	0.10%	0.17%	0.36%	0.76%	0.44%
LI (short)	0.27%	0.37%	0.28%	0.14%	0.19%	0.47%	0.62%	0.31%
LI (total)	0.39%	0.71%	0.83%	0.23%	0.35%	0.83%	1.38%	0.75%

- More frequent liquidations on USDT direct perpetuals
- Most frequent liquidations on ByBit
- Contrasts previous results using OI in denominator

How Much Leverage?

Long Position on Direct Perpetual

- Suppose leverage λ used to open long position when perpetual price is $F_0 \Rightarrow$ initial margin IM is $\frac{F_0}{\lambda}$
- Assume no additional collateral in/out of margin account
- Maintenance margin rate is $\frac{1}{2}$ initial margin rate $\Rightarrow \frac{1}{2\lambda}$
- Collateral in margin account at time τ is IM $-$ MM$_\tau = \frac{F_0}{\lambda} - \frac{F_\tau}{2\lambda}$
- Loss on long position at time τ is $F_0 - F_\tau$
- Auto-liquidation occurs at time T if loss exceeds collateral, i.e. when:

$$F_0 - F_\tau > \frac{F_0}{\lambda} - \frac{F_\tau}{2\lambda}$$

- Implied leverage is minimum value of λ to prevent auto-liquidation
- Found by setting equality above, $\Rightarrow \lambda = \frac{F_\tau}{2(F_0 - F_\tau)} + 1$

Implied Leverage

Long/Short Positions on Direct/Inverse Perpetual

- Similar arguments for short positions on direct d perpetuals, and long l or short s positions on inverse perpetuals i yield the following:

$$\lambda^d_{lt} = \frac{F_T}{2(F_t - F_T)} + 1,\ \lambda^s_{lt} = \frac{F_T}{2(F_T - F_t)} - 1$$
$$\lambda^i_{lt} = \frac{F_t}{2(F_t - F_T)} - 1,\ \lambda^i_{st} = \frac{F_t}{2(F_T - F_t)} + 1$$

Average implied leverages over all 4-hr internals from 1 Jan to 31 May 2021

	USD Inverse Perpetuals					USDT Direct Perpetuals		
	Binance	BitMEX	Bybit	Deribit	OKEx	Binance	Bybit	OKEx
λ_l	11.26	8.28	12.53	7.04	8.71	12.55	14.16	13.51
λ_s	14.95	14.38	15.23	6.70	15.17	12.64	12.61	10.41

- Implied leverage like average leverage of all traders on exchange
- 7X –15X is maximum leverage for positions sizes about 100 BTC (inverse)
- 10X – 14X is maximum leverage for positions sizes about $15 million (direct)

How Aggressive are Auto-Liquidations on Different Exchanges?

$$\text{AI} = \text{Aggressive Index} = \frac{\sum_t [\lambda_{lt}\text{LLV}_t + \lambda_{st}\text{SLV}_t]}{\sum_t \text{TV}_t}$$

- TV$_t$ is total volume traded during time interval t
- LLV$_t$ and SLV$_t$ are total size of long and short auto-liquidations
- Aggregated over 4-hr intervals
- Also decomposed into long and short indices....

Aggressive Indices for Period 1 Jan and 31 May 2021

	USD Inverse Perpetuals					USDT Direct Perpetuals		
	Binance	BitMEX	Bybit	Deribit	OKEx	Binance	Bybit	OKEx
AI (long)	3.52%	4.76%	9.03%	1.93%	3.22%	6.25%	13.12%	5.82%
AI (short)	1.52%	4.37%	3.65%	1.61%	2.41%	3.63%	8.81%	5.63%
AI	5.04%	9.13%	12.69%	3.54%	5.63%	9.89%	21.92%	11.45%

- Most aggressive liquidations are on Bybit, especially long positions

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